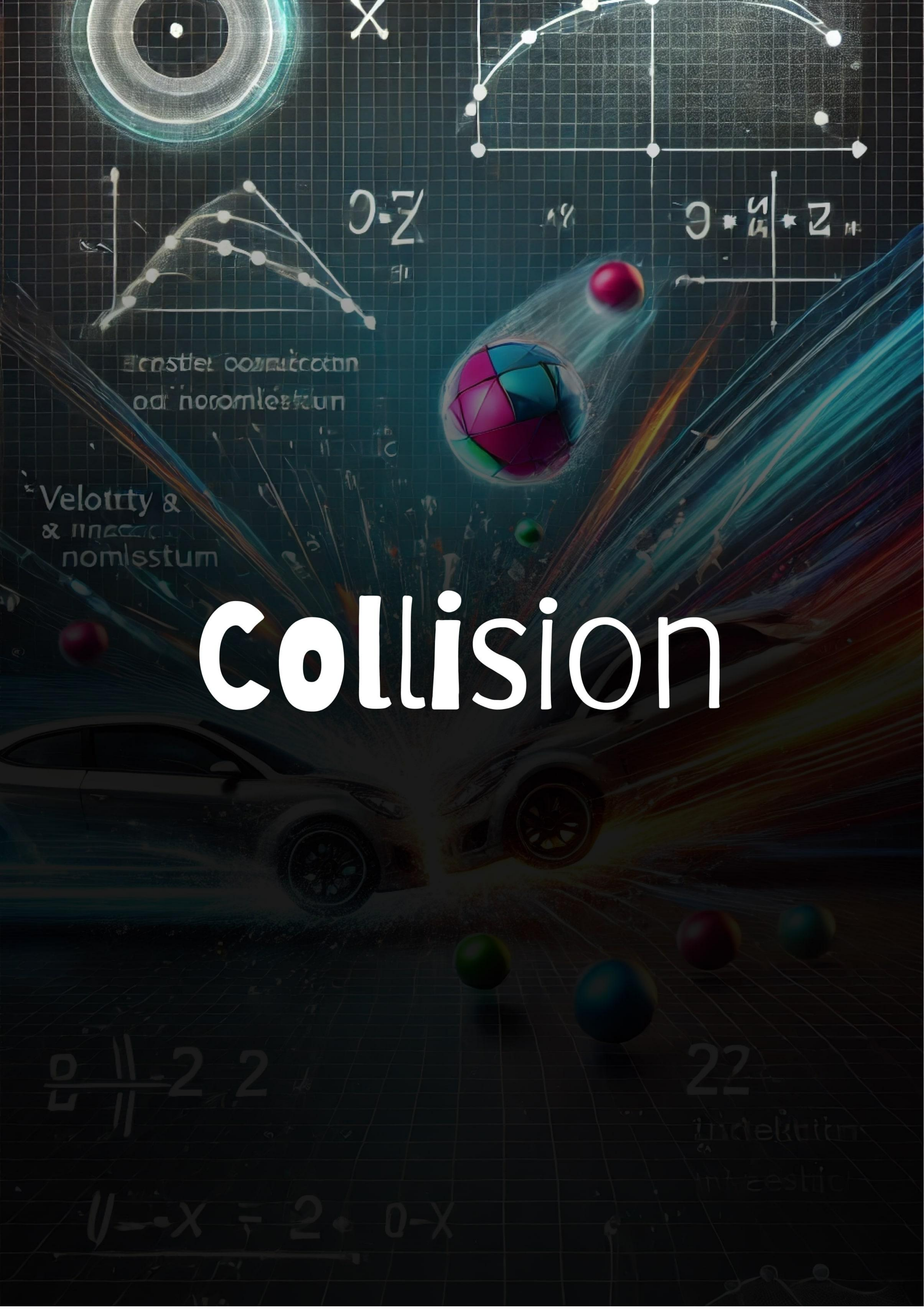


Collision

Collissions

LINE OF IMPACT

The common normal drawn at the point of impact b/w two colliding surfaces is called line of impact.

VELOCITY OF APPROACH

The relative velocity of the colliding points just before Collision along the line of impact is called velocity of approach.

$$\boxed{V_a = V_{1N} - V_{2N}}$$

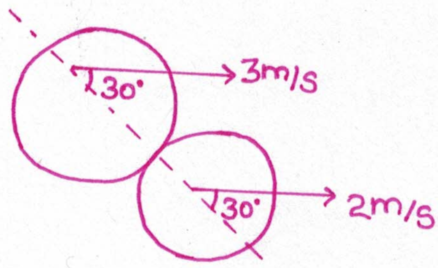
EXAMPLE :

(1)

$(u_1 > u_2)$

$V_a = u_1 - u_2$

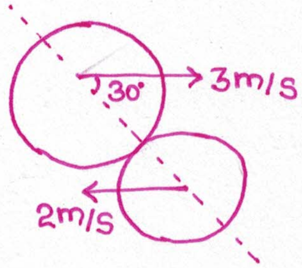
(2)



$V_a = u_1 \cos 30^\circ - u_2 \cos 30^\circ$

$= \frac{\sqrt{3}}{2}$

(3)

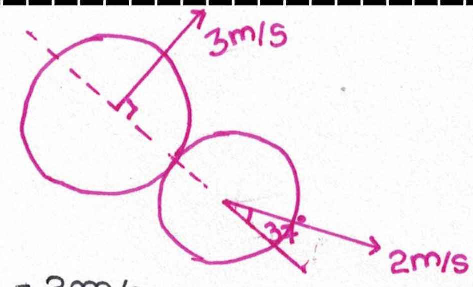


$V_a = 3 \cos 30^\circ + 2 \cos 30^\circ$

$= \frac{5\sqrt{3}}{2}$

VELOCITY OF SEPARATION

The relative velocity of the colliding points just after the collission along the line of impact is called velocity of separation.



$V_S = 2\,m/s$

$V_S = V_{2N} - V_{1N}$

COEFFICIENT OF RESTITUTION (COR)

The ratio of velocity of separation to the velocity of approach during the collision is called the coefficient of restitution.

Mathematically,
$$e = \frac{\vec{V_S}}{\vec{V_a}} = \frac{V_{2N} - V_{1N}}{U_{1N} - U_{2N}}$$

$$V_{2N} - V_{1N} = e\,(U_{1N} - U_{2N})$$

For collission b/w smooth spheres, we can classify the collissions based on:

⇒ If $e = 1$, we say it is perfectly elastic collission.

⇒ If $e = 0$, we say it is perfectly inelastic collission.

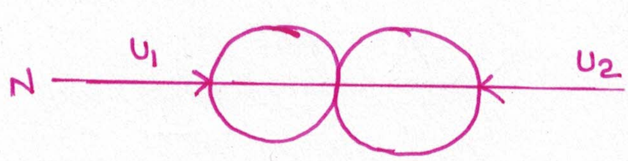
⇒ If $0 < e < 1$, we say it is inelastic collission.

CLASSIFICATION OF COLLISSIONS

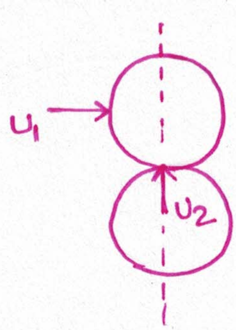
(a) BASED ON LINE OF IMPACT

(1) If during collission, velocities of both the bodies before collission are along the line of impact, then we call it head on collission, or direct collission.

(2) If the velocity of atleast one of the bodies is not along the line of impact before collission, then we call it oblique collission or indirect collission.



Direct / Head-on Collission



Oblique collision

(b) BASED ON ENERGY CONSIDERATIONS

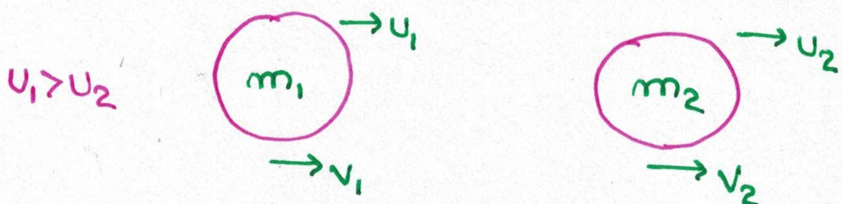
If the mechanical energy in a collission is conserved, we call it elastic collission.
If the mechanical energy in a collission is not conserved, we call it inelastic collission.

(C) BASED ON NATURE OF CONSTRAINT

(1) **Free Collission:** If during collission, there is no impulse on the colliding bodies apart from the impulse of mutual collission, we call it free collission.

(2) **Constraint Collission:** If during collission, there are impulses on the colliding bodies apart from the impulse of mutual collission (J_T & J_G), we call it constraint collission.

Que.) Show that if the mechanical energy is conserved in a collission b/w smooth spheres then $e = 1$



COM

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad (F_{ext} = 0)$$
$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad — (1)$$

COME

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$
$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad — (2)$$

(2) ÷ (1)

$$(u_1 + v_1) = u_2 + v_2$$
$$v_2 - v_1 = u_1 - u_2$$
$$\frac{v_2 - v_1}{u_2 - u_1} = e = 1$$

NOTE: We can call any body as (1) or (2) and any directions of line of impact as positive but we should not change it after collission.
In most cases we make two eqn. of COR and COM.

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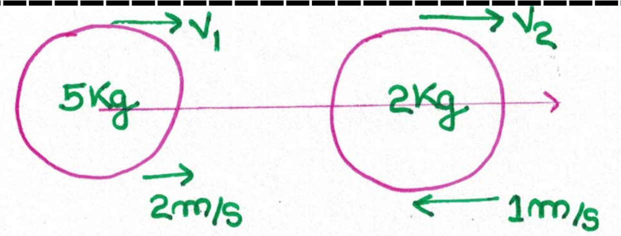
Que.)



$e = \frac{1}{2}$

COR

$$v_2 - 2v_1 = \frac{3}{2} \quad —①$$

COM

$$5\times 2 - 2 = 5v_1 + 2v_2$$

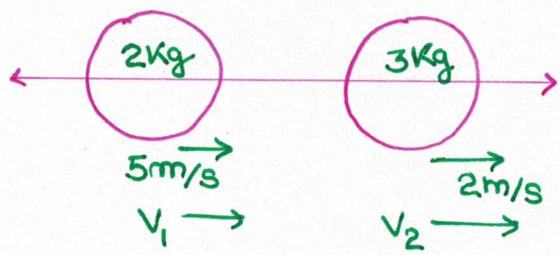
$$8 = 5v_1 + 2v_2 \quad —②$$

$$7v_1 = 5$$

$$v_1 = \frac{5}{7} \quad , \; v_2 = \frac{3}{2} + \frac{5}{7}$$

$$v_2 = \frac{31}{14}$$

Que.)



$e = \frac{1}{3}$

COR $\quad v_2 - v_1 = \frac{1}{\cancel{3}} \times \cancel{3} = +1 \quad —①$

COM $\quad 10 + 6 = 2v_1 + 3v_2$

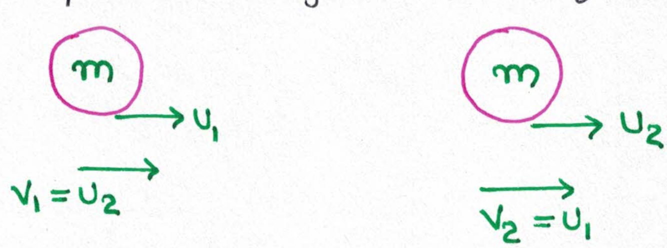
$$16 = 2v_1 + 3v_2$$

$$5v_2 = 18$$

$$v_2 = \frac{18}{5}$$

SPECIAL CASES IN FREE COLLISIONS

(1) In perfectly elastic collission b/w bodies of identical mass, the velocity component along the line of impact gets exchanged.



PROOF

COM

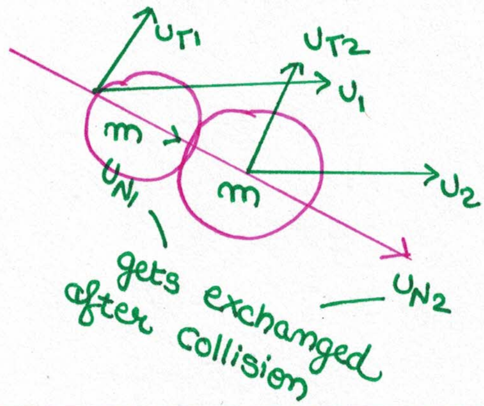
$$mu_1 + mu_2 = mv_1 + mv_2$$

$$u_1 + u_2 = v_1 + v_2 \quad —①$$

COR $\quad u_1 - u_2 = v_2 - v_1 \quad$ — ②

$2u_1 = 2v_2$

$u_1 = v_2 \quad \& \quad u_2 = v_1$

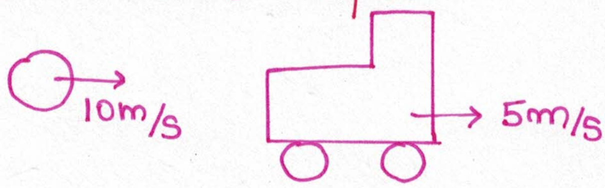


NOTE: In free oblique collisions, the tangential velocities remain unchanged irrespective of coefficient of restitution & masses of smooth balls.

(2) If one of the bodies, has a very large mass as compared to the other body, we can assume that velocity massive body remains unchanged.

(In such cases we only need to make the COR eqⁿs)

Que.) A naughty boy throws a tennis ball on a truck to make a perfectly elastic head on impact.



COR

$5 = 5 - v_2$

$v_2 = 0$

If truck

$15 = -v - 5$

$20 = -v$

$v = -20$

(3) If $e = 0$, then we only need COM eqⁿ because the bodies stick together in case of head on collisions.

FREE OBLIQUE COLLISIONS

In case of oblique collisions, we resolve all the velocities along the line of impact and $\perp$ to the line of impact.

Say, u_{1N} , u_{2N}

u_{1T} , u_{2T}

v_{1N} , v_{2N}

v_{1T} , v_{2T}

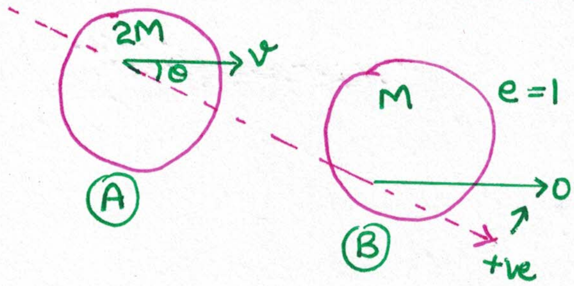
then we can make following eq^{n}s :

(1) COM along line of impact.

(2) COR

(3) $v_{1T} = u_{1T}$, $v_{2T} = u_{2T}$ (there is no impulse along tangential direction)

Que.) Find post collision speed of balls.



$v\cos\theta = -v_{AN} + v_{BN}$ (COR)

$2Mv\cos\theta = Mv_{BN} + 2Mv_{AN}$ (COM)

$2v_{AN} + v_{BN} = 2v\cos\theta$

$v\cos\theta = 3v_{AN}$

$v_{AN} = \dfrac{v\cos\theta}{3}$, $v_{BN} = \dfrac{4v\cos\theta}{3}$

$v_A = \dfrac{v\cos\theta}{3}\hat{i} + v\sin\theta\hat{j}$

$v_B = \dfrac{4v\cos\theta}{3}\hat{i}$

CONSTRAINT COLLISION

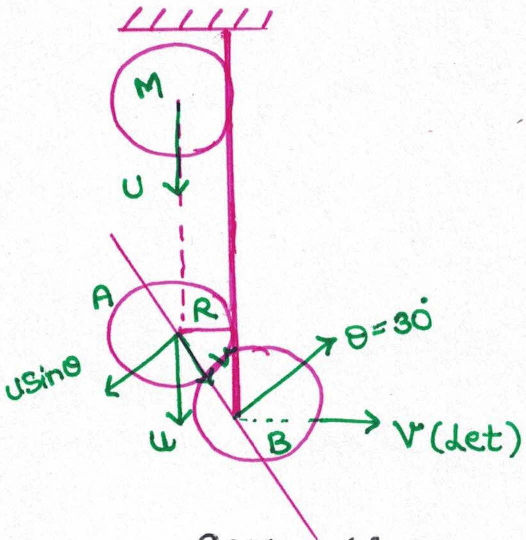
In constraint collision, we would usually make the following eq^{n}s

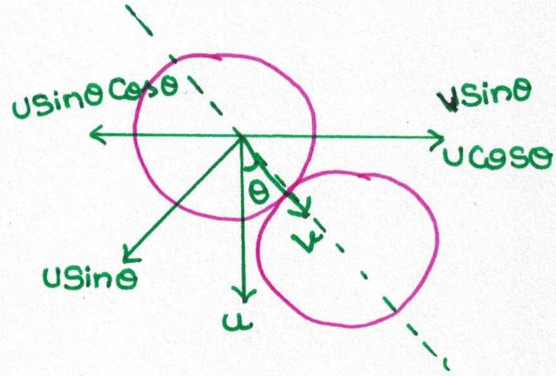
(1) COM in direction where there is no external impulse.

(2) COR along the line of impact.

(3) If required, $\angle$IM in the direction of external impulse. (This will be required if we need to solve for external impulse).

Que.) In the system shown, ball A hits an identical ball B with a speed u vertically down. If the coefficient of restitution is e, develop a system of eqn. to solve for V (velocity of ball B), v (Component of ball A) & J_T & J_N.

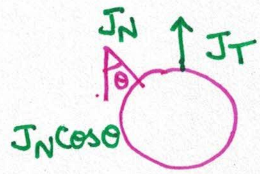




COM (horizontal)

$$MV + M(v\sin\theta - u\sin\theta.\cos\theta) = 0$$

COR_N $\quad v\sin\theta - V = e(u\cos\theta)$

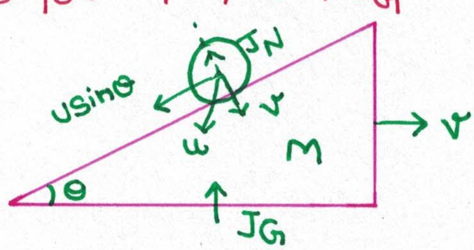


$LIM_B\uparrow$ $\quad J_T = J_N\cos\theta$

$LIM_B\rightarrow$ $\quad J_N\sin\theta = mV$

$LIM_B\rightarrow$ $\quad J_N\sin\theta = mV$

Que.) Solve for V, v, J_N, J_G



COR = e

COM $\longleftrightarrow$

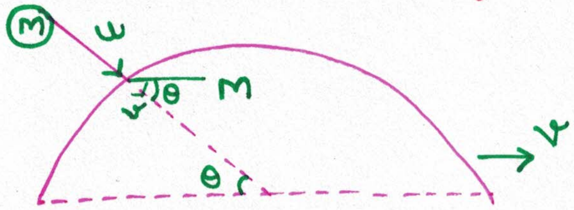
$$M(v\sin\theta - u\sin\theta.\cos\theta) + MV = 0$$

COR $\quad v\sin\theta - V = e(u\cos\theta)$

$LIM_B\uparrow$ $\quad J_N = -mv + mu\cos\theta$

$LIM_W\uparrow$ $\quad J_N\cos\theta = J_G$

Que.) Solve for post collision velocities of shell & ball.

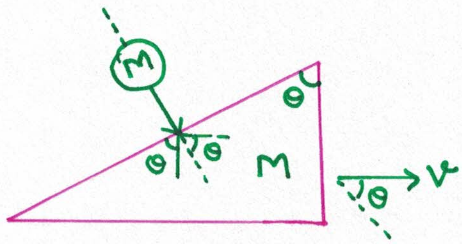


$e = 1$

$\underline{\text{COM}} \leftrightarrow \quad Mv + Mv\cos\theta = Mu\cos\theta$

$\underline{\text{COR}} \leftrightarrow \quad v\cos\theta - v = u$

Que.)



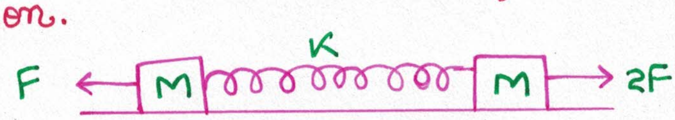
$e = 1$

$\underline{\text{COM}} \quad Mv + Mv\cos\theta = Mu\cos\theta$

$\underline{\text{COR}} \quad v\cos\theta - v = u$

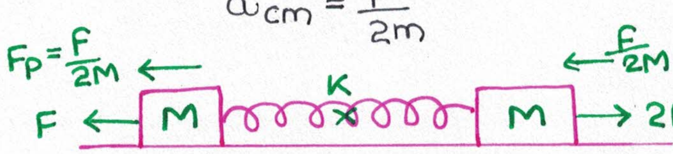
PROBLEMS INVOLVING CENTRE OF MASS FRAME

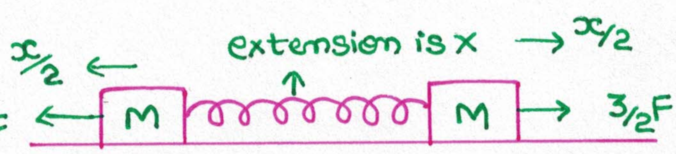
Que.) Two blocks of mass M each are connected with a string of constant K on a smooth horizontal surface. The system is at rest. Now we apply 2 forces on the blocks as shown. Find the max. extension of the spring in the subsequent motion.



visualising from centre of mass frame

$$a_{cm} = \frac{F}{2m}$$





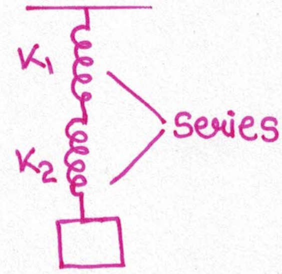
$\underline{\text{CHIME}}$

$$\frac{3F}{\cancel{2}} \times \frac{x}{\cancel{2}} \times \cancel{2} = \frac{1}{\cancel{2}} Kx^{\cancel{2}}$$

$$x = \frac{3F}{K}$$

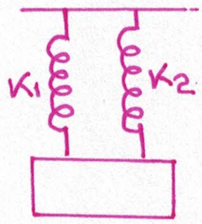
NOTE : (1) Springs in series, then

$$\frac{1}{K_{eff}} = \frac{1}{K_1} + \frac{1}{K_2}$$

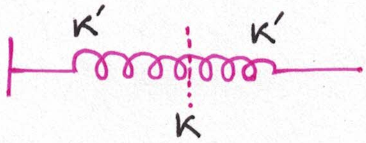


(2) Springs in parallel, then

$$K_{eff} = K_1 + K_2$$



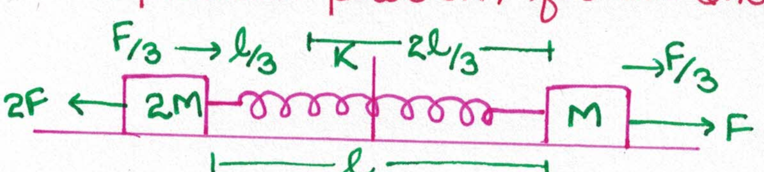
Que.) Prove that the spring constant becomes double when spring is half.



$$\frac{1}{K} = \frac{1}{K'} + \frac{1}{K'}$$

$$2K = K'$$

Que.) Repeat the previous problem for the shown case.



$$a_{cm} = \frac{F}{3M}$$

CHIME

$$\frac{4}{3}F \times \frac{\cancel{x}}{3} + \frac{4}{3}F \times \frac{2\cancel{x}}{3} = \frac{1}{2}Kx^{\cancel{2}}$$

$$x = \frac{16F}{9K}$$

Que.) Two balls are connected on a smooth horizontal surface as shown. Now the velocity v_0 is given to ball B. Find (i) v_{cm}

(ii) Velocities of the spheres after the string has rotated through 90°.

(iii) Tension in the connected string.

(i) $$v_{cm} = \frac{\cancel{M}v_0}{2\cancel{M}} = \frac{v_0}{2}$$

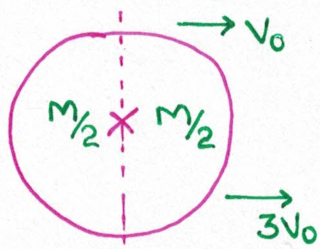
(ii) $$v_A = v_B = \sqrt{\left(\frac{v_0}{2}\right)^2 + \left(\frac{v_0}{2}\right)^2}$$

$$v_A = v_B = \frac{\sqrt{2}\,v_0}{2}$$

(iii) $$T = \frac{M\left(v_0/2\right)^2}{l/2}$$

$$= \frac{Mv_0^2}{2l}$$

Que.) A bomb moving with speed v_0 splits into two parts such that velocity of the front part becomes $3v_0$ as shown. How much energy is released during v_0.
(Analyse the problem from Centre of mass frame).



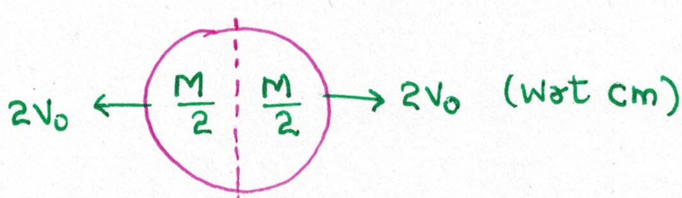
$V_{cm} = V_0$

V_{cm} w.r.t. cm = 0

W.r.t. cm

$M\,V_{cm} = 0$

COM

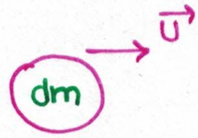


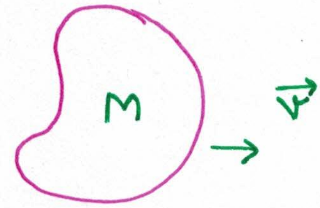
∴ Change in K.E.

$$= \frac{1}{2} \times \frac{M}{2} \times (2v_0)^2 \times 2$$

$$= 2v_0^2 M$$

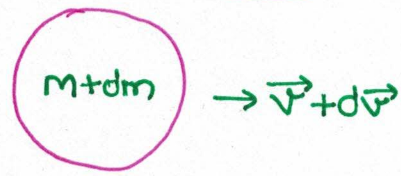
VARIABLE MASS SYSTEM





$\vec{P}_f = (M+dm)(\vec{v}+d\vec{v})$

$\vec{P}_i = M\vec{v} + dm\,\vec{u}$



$d\vec{P} = \vec{P}_f - \vec{P}_i$

$= (M+dm)(\vec{v}+d\vec{v}) - M\vec{v} - dm\,\vec{u}$

$= dm\,\vec{v} + M\,d\vec{v} + \underbrace{dm\,dv}_{\text{can be neglected}} - dm\,\vec{u}$

$\therefore d\vec{p} = M\,d\vec{v} + dm\,(\vec{v}-\vec{u})$

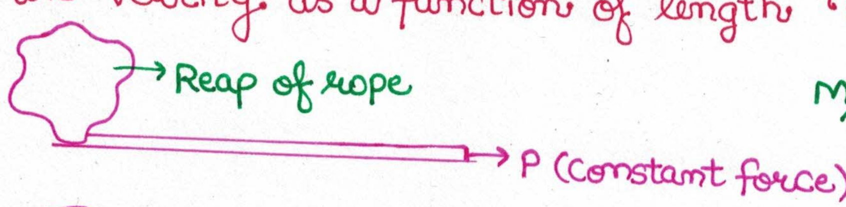
$$\frac{d\vec{p}}{dt} = M\frac{d\vec{v}}{dt} + (\vec{v}-\vec{u})\cdot\frac{dm}{dt}$$

$$\boxed{F_{ext} = M\frac{d\vec{v}}{dt} + (\vec{v}-\vec{u})\frac{dm}{dt}}$$

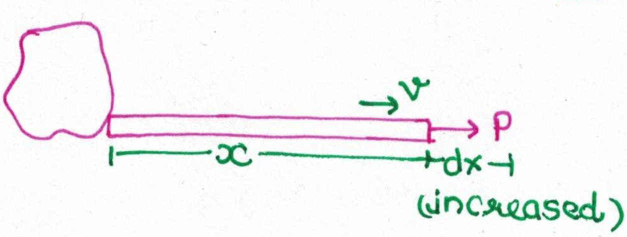
$$\frac{M\,d\vec{v}}{dt} = \vec{F}_{ext} + \vec{v}_{rel}\cdot\frac{dm}{dt}$$

or $$\frac{M\,d\vec{v}}{dt} = \vec{F}_{ext} + (\vec{u}-\vec{v})\frac{dm}{dt}$$

Que.) Find the velocity as a function of length 'x' of rope.



$M/\ell = \lambda$



$\vec{u} = 0$

$\vec{v}$ (let)

(in x-direction)

$$\frac{dm}{dt} = \lambda\frac{dx}{dt} = \lambda v$$

$$\frac{m\,dv}{dt} = P - \lambda v^2$$

$$\lambda x\,\frac{dv}{dx}\times\frac{dx}{dt} = P - \lambda v^2$$

$$\lambda x\,v\frac{dv}{dx} = P - \lambda v^2$$

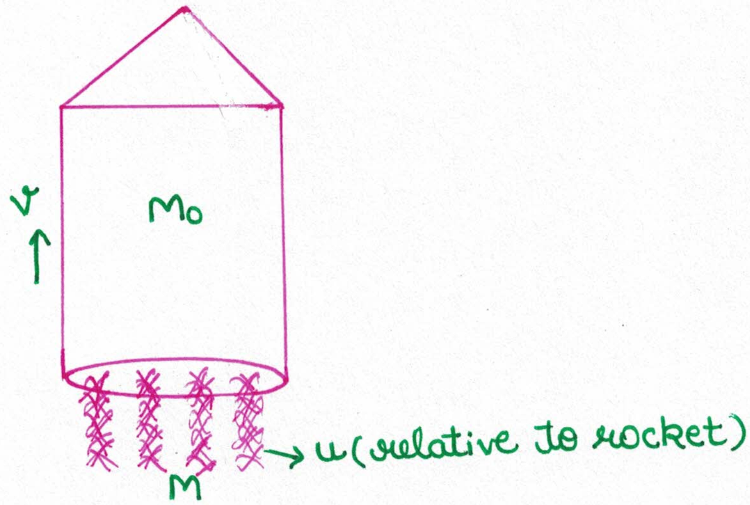
$$\int \frac{v.dv}{P-\lambda v^2} = \int \frac{dx}{\lambda x}$$

Let $z = P-\lambda v^2$

$dz = 0 - 2\lambda v dv$

$vdv = -\frac{dz}{2\lambda}$

Que:)



$M_0 - M = m$

$\frac{dm}{dt} = \frac{dM}{dt} = -\alpha$

$\frac{md\vec{v}}{dt} = -mg - u(-\alpha)$